ANTIPARTICLES AS BOUND STATES OF PARTICLES IN THE FACTORIZED S-MATRIX FRAMEWORK

V. KURAK and J.A. SWIECA
Departamento de Física, Pontifícia Universidade Católica, Rio de Janeiro, RJ, Brasil

Received 29 November 1978

We show that in a factorized two-dimensional S-matrix having SU(N) ($N > 2$) symmetry the antiparticles (transforming according to $\{\bar{N}\}$) are bound states of particles and vice versa and argue that this S-matrix is the one of the chiral Gross–Neveu model with screened U(1) charge and pseudocharge.

Since Zamolodchikov [1] wrote down the exact S-matrix of the quantum sine-Gordon solitons a large number of two-dimensional models [2–6] have been understood. One usually refers to the corresponding exact S-matrices as factorized owing to the fact that the elastic multiparticle amplitude is a product of the two-particle one, production being absent.

The chiral Gross–Neveu model [7] defined by the lagrangian

$$\mathcal{L} = \mathrm{i} \sum_{i=1}^{N} \bar{\psi}_i \not{\partial} \psi_i + \tfrac{1}{2} g^2 \left[\left(\sum_{i=1}^{N} \bar{\psi}_i \psi_i \right)^2 - \left(\sum_{i=1}^{N} \bar{\psi}_i \gamma^5 \psi_i \right)^2 \right], \tag{1}$$

was suspected to belong to this class of models for a long time, the main arguments coming from the similarity to the ordinary Gross–Neveu model and from classical [8,9] and quasi-classical [10] considerations. But the $1/N$-expansion used in ref. [7] shows spontaneous breaking of the U(1) chiral symmetry and the associated Goldstone boson which of course cannot exist in two-dimensional space–time [11].

In a recent paper [12] it was argued that one can reconcile spontaneous mass generation with the absence of spontaneous symmetry breakdown in this model. This is most easily seen in terms of the boson representation for the fermion fields [13], where ψ is written in terms of exponentials of the U(N) currents potentials.

As a result of U(1) × U(1) symmetry one of those potentials is a zero mass free field (say ϕ). This means that the field ψ will describe infraparticles [14]. In order to extract the real particle content of this model one removes the exponentials of the unwanted massless excitation obtaining a new field $\hat{\psi}$ which does not carry either U (1) chirality or the U(1) charge [‡1]. This field is expected to describe massive particles and transforms according to SU(N). The Green's functions of ψ are products of an explicitly computable factor involving only the field ϕ times the Green's functions of $\hat{\psi}$.

As a result of charge screening the field $\hat{\psi}$ will satisfy the following identity (up to a Klein transformation factor):

$$\hat{\psi}_i^+ = \frac{1}{(N-1)!}\, \epsilon_{ij_1 \dots j_{N-1}} \hat{\psi}_{j_1} \hat{\psi}_{j_2} \dots \hat{\psi}_{j_{N-1}} \, . \tag{2}$$

In an interacting model with particle content this means that the antiparticles (belonging to $\{\bar{N}\}$) are bound states of $N-1$ particles and vice versa. Furthermore, from the bosonized form of this model one sees that there is a conjugation symmetry which implies that the mass of those bound states coincides with the mass of the original particles. It is apparent from eq. (2) that the fields $\hat{\psi}$ satisfy neither bose nor fermi statistics but correspond rather to the massive version

‡1 The latter property does not follow from the method used in ref. [12] but is obvious from bosonization.

of the fields introduced in ref. [15]. Since they are non-local in the usual sense they provide an illustration of Carruthers theorem [16].

In ref. [9] it was attempted for the first time to construct the exact S-matrix of the model (1) using the original U(N) symmetry.

If this model really has a factorized S-matrix one has to face the problem of formulating a scattering problem having SU(N) and not U(N) symmetry. The strategy to construct this S-matrix is to realize that the fact that antiparticles are bound states of particles allows us to transform this problem into a U(N) one provided there is in the $\{\bar{N}\}$ channel a bound state of $N-1$ particles with the same mass as the original particle. Consistency further requires that the scattering amplitudes of those bound states coincide with the scattering amplitudes of the original particles.

The U(N) problem was solved in ref. [17] to where we refer the reader for details. One introduces particles $\alpha(\theta)$ and antiparticles $\bar{\alpha}(\theta)$ transforming according to $\{N\}$ and $\{\bar{N}\}$ of U(N) where θ is the rapidity variable related to the energy and momentum by $p_0 = m\mathrm{ch}\,\theta$, $p_1 = m\mathrm{sh}\,\theta$, m is the common mass of the particles and antiparticles. In ref. [17] it was shown that six classes emerge as solutions of the requirement of factorization. Since we have in mind to construct the particle–antiparticle amplitude from N-particle scattering and since from factorization in this case it follows that reflection is impossible it is clear that class II of ref. [17] is the right candidate. In this case one has

$$\langle\delta(\theta_2)\gamma(\theta_1)|\alpha(\theta_1)\beta(\theta_2)\rangle = u_1(\varphi)\delta_{\alpha\gamma}\delta_{\beta\delta} + u_2(\varphi)\delta_{\alpha\delta}\delta_{\beta\delta}\,,$$

$$\langle\bar{\delta}(\theta_2)\gamma(\theta_1)|\alpha(\theta_1)\bar{\beta}(\theta_2)\rangle = t_1(\varphi)\delta_{\alpha\gamma}\delta_{\beta\delta} + t_2(\varphi)\delta_{\alpha\beta}\delta_{\gamma\delta}\,,$$

where $\varphi = (\theta_1 - \theta_2)/\mathrm{i}\pi$ and

$$t_1(\varphi) = \frac{\Gamma(1/2+\varphi/2)\,\Gamma(1/2-\lambda/2-\varphi/2)}{\Gamma(1/2-\varphi/2)\,\Gamma(1/2-\lambda/2+\varphi/2)}\,,\qquad t_2(\varphi) = [-\lambda/(1-\varphi)]\,t_1(\varphi)\,, \tag{3}$$

$$u_1(\varphi) = \frac{\Gamma(1-\varphi/2)}{\Gamma(1-\varphi/2-\lambda/2)}\,\frac{\Gamma(\varphi/2-\lambda/2)}{\Gamma(\varphi/2)}\,,\qquad u_2(\varphi) = (-\lambda/\varphi)u_1(\varphi)\,, \tag{4}$$

with $\lambda = 2/N$. Notice the difference between eq. (3) and formula (17) of ref. [17]. We have replaced λ by $-\lambda$ in eqs. (3,4) such that we have introduced a bound state in the particle–particle antisymmetric channel $(u_1 - u_2)$ with mass

$$m_2 = m\sin(2\pi/N)/\sin(\pi/N)\,, \tag{5}$$

and excluded the symmetric one $(u_1 + u_2)$.

To simplify matters let us first exemplify our calculation considering the case $N = 3$. Computing the residuum of the amplitude (at the pole $\theta_{12} = \mathrm{i}\pi\lambda$)

$$\begin{aligned}&2^{-1/2}\epsilon_{\hat{\Sigma}\hat{\alpha}\hat{\beta}}2^{-1/2}\epsilon_{\Sigma\beta\alpha}\\ &\quad\times\langle\hat{\gamma}(\theta_3)\hat{\beta}(\theta_2)\hat{\alpha}(\theta_1)|\alpha(\theta_1)\beta(\theta_2)\gamma(\theta_3)\rangle\\ &\quad= -\tfrac{1}{2}(u_1(\theta_{12}) - u_2(\theta_{12}))\\ &\quad\times\{[2u_1(\theta_{13})u_1(\theta_{23}) + u_1(\theta_{13})u_2(\theta_{23})\\ &\quad+ u_2(\theta_{13})u_1(\theta_{23}) - u_2(\theta_{13})u_2(\theta_{23})]\,\delta_{\Sigma\hat{\Sigma}}\delta_{\gamma\hat{\gamma}}\\ &\quad- [u_1(\theta_{13})u_2(\theta_{23}) + u_2(\theta_{13})u_1(\theta_{23})\\ &\quad- u_2(\theta_{13})u_2(\theta_{23})]\,\delta_{\Sigma\gamma}\delta_{\hat{\Sigma}\hat{\gamma}}\}\,,\end{aligned}$$

which we will call

$$\langle\gamma(\theta_2)\,\hat{\bar{\Sigma}}(\theta_1)|\bar{\Sigma}(\theta_1)\gamma(\theta_2)\rangle = T_1(\varphi)\delta_{\Sigma\hat{\Sigma}}\delta_{\gamma\hat{\gamma}} + T_2(\varphi)\delta_{\Sigma\gamma}\delta_{\hat{\Sigma}\hat{\gamma}}\,, \tag{6}$$

one gets $T_1(\varphi) = t_1(\varphi)$, $T_2(\varphi) = t_2(\varphi)$. The bound state $\bar{3}$ that we have denoted by the capital letter $\bar{\Sigma}$ must therefore be identified with the original antiparticle. In this way one looses the U(1) part of U(3) and this leads to a genuine SU(3) S-matrix.

For the general SU(N) case two points should be noticed:

(a) As a consequence of two-particle bound state (5) it follows from general arguments [18] that there is an n-particle bound state with mass

$$m_n = \frac{m\sin(n\pi/N)}{\sin(\pi/N)}\,,\qquad n = 1, \ldots, N-1\,.$$

For $N-1$ particles in the $\{\bar{N}\}$ channel one has therefore $m_{N-1} = m$.

(b) Proceeding as in the SU(3) case one easily finds that

$$T_1(\varphi) = \frac{P_1(\varphi)}{Q_1(\varphi)}\,t_1(\varphi)\,,\qquad T_2(\varphi) = \frac{P_2(\varphi)}{Q_2(\varphi)}\,t_2(\varphi)\,,$$

where $P_{1,2}(\varphi)$ and $Q_{1,2}(\varphi)$ are polynomials. Since $T_1(\varphi)$, $T_2(\varphi)$ are by themselves class II scattering amplitudes it follows from ref. [17] that $T_1(\varphi) = t_1(\varphi)$,

$T_2(\varphi) = t_2(\varphi)$, which proves again the desired result that the $\{\bar{N}\}$ bound state Σ is the antiparticle of the original particle.

We remark that using the identification provided by eq. (2) one finds that class II [17] for $N = 2$ also leads to a SU(2) symmetry. As noted in ref. [6] the correspondence with the sine-Gordon solitons is $t_1 = t - r$, where t and r are the transmission and reflection amplitudes of the soliton–antisoliton scattering.

In a future paper we plan to improve our confidence that this S-matrix belongs to the chiral Gross–Neveu model by an explicit perturbation check.

V.K. thanks for discussions with Peter Weisz. We acknowledge several discussions with Bert Schroer.

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